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## Study of Optimum Discrete Estimators in Measurement Analysis

A survey study, which is presented in a report, has been made of statistical techniques (statistical filtering and prediction) for obtaining the best estimate of the true data parameters from discrete measured quantities containing random errors. The equations studied are developed in the time domain and lead to a digital computer realization of the optimal filter, as opposed to the frequency domain approach in which the optimal filter may be conveniently obtained through network synthesis. The state variable notation is used throughout the study. Unknown parameters, such as position and velocity of a space vehicle, are grouped into a composite state vector which represents the information necessary to specify the process in phase space. Using these techniques, estimation procedures are developed as an iterative algorithm for digital computation in real time.

Chapter II of the report contains the mathematical foundations which are used extensively in the subsequent chapters. The basic concepts of probability theory and random processes are discussed. The state space formulation of dynamic systems is also introduced. The concept of dynamic programming is presented and two very important matrix identities which are used extensively in the remaining chapters are introduced. An expression for the gradient of a general quadratic form is also given.

Chapter III considers the static problem in which the measurements of the process are linearly transformed state variables corrupted by random measurement noise. A modern sequential technique for processing the measurements in an optimal manner is presented. This modern technique is compared with a recursive version of the classical sample average approach. The conditions under which the two techniques are identical are also discussed and examples are given to illustrate the methods. The optimum estimation equations presented in this Chapter are derived in Appendix A.

In Chapter IV, the techniques of Chapter III are generalized to the case where the process dynamics are known and given by a vector difference equation. Recursive relationships for processing the measurements in an optimal fashion are obtained. A dynamic version of the sample average technique is fabricated and compared with the modern approach. Generalizations of the approach to include problems of trajectory estimation, correlated errors, etc. are discussed. The optimum estimation equations presented in this Chapter are obtained, using a variety of techniques, in Appendix B. The trajectory estimation equations are derived in Appendix C.

In Chapter V, all the available techniques for estimating a nonlinear process are reviewed. Both the discrete and continuous cases with known and unknown process dynamics are considered. Some of the techniques which are closely related to those of the previous chapter are derived in detail in Appendix D.

## Note:

Copies of the report may be obtained from: Technology Utilization Officer

Marshall Space Flight Center Huntsville, Alabama 35812 Reference: B68-10348

## Patent status:

No patent action is contemplated by NASA.

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